Economics

The Behavioral Model for Estimating the Laffer Fiscal Points

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ABSTRACT. The article examines the approach to estimating the effect of the tax burden on the amount of total output and budget revenues. This approach uses a behavioral model, with a specific version of an entropy function. In the context of production technology, to quantitatively estimate the dependence of output on the amount of the tax burden, the article reflects the expansions of the macroeconomic production function in which the role of the average tax rate is distinguished in some form. The suggested model makes it possible to determine the so-called fiscal points corresponding to the maximum production effect and the budget’s maximum tax revenues. The conclusion is drawn that, these points correspond to the Laffer concept, since for the points of the behavioral model the amount of use of economic resources occurs endogenously. The results obtained are illustrated using existing data on the U.S. economy. When different versions of the calculations were carried out, the estimated model as a whole, as well as its parameters, maintained its stability and did not lose its statistical significance in a fairly broad range of changes in the “sample size.” Even when the quality of the model deteriorated (the parameters being estimated became statistically insignificant) as a result of excessive reduction of the sample size, the estimates of the fiscal characteristics changed only slightly. This is not sufficient grounds for drawing final conclusions about the suitability of the suggested model for conducting specific applied calculations. © 2013 Bull. Georg. Natl. Acad. Sci.

Key words: average tax rate, production technology, production function, entropy function, Balatski fiscal points, Laffer fiscal points, optimal tax rate, potential level of output.
budget revenues according to the change in the involvement of resources in production. Both of these effects can be analyzed and estimated based on mathematical economic models.

Two such models are presented in this article. In one model, the tax burden (average tax rate) is a factor determining the technology and efficiency of resource use, while in the other it is a factor determining the amount of resource use and the level of economic activity. Both of these models consider the values of total output and budget revenues as functions depending on the aggregated tax rate. If total output is designated as \( Y \), and the budget’s tax revenues as \( T \), then we can write

\[
Y(t) = Y(t) \quad \text{and} \quad T(t) = T(t),
\]

where \( t \) is the aggregated (average) tax rate (the ratio of the budget’s total tax revenues to GDP), which satisfies the condition \( 0 \leq t \leq 1 \). In this case, it is understood that the functions \( Y(t) \) and \( T(t) \) are interrelated as \( T(t) = tY(t) \). This relationship shows that the behavior of the budget revenues function is substantially determined by the behavior of \( Y(t) \). Therefore, of these two functions more attention will be given to the total output function \( Y(t) \).

In the context of production technology, to quantitatively estimate the dependence of output on the amount of the tax burden, we can use expansions of the macroeconomic production function in which the role of the average tax rate is distinguished in some form. Such an expansion is possible in two basic directions. In one direction, taxes should be seen as a component of production technology. In the second way of expanding the production function, taxes are seen not as components of technology, but as factors that act on the efficiency of technology, or rather, on the efficiency of the resources used in technology: labor and capital. One version of such an expansion is suggested by Evgeny Balatsky, it is a production function with variable elasticity, in the following form [3: 88]:

\[
Y(t) = \gamma DK^{\alpha(t)}N^{\beta(t)}, \quad (1)
\]

where \( K \) is the cost of the capital used; \( N \) is the amount of labor used; \( D \) is a trend operator (a function of which the argument is time); \( \alpha(t) \) and \( \beta(t) \) are the capital and labor elasticity coefficients of output, the values of which depend on the average tax rate \( t \); and \( \gamma \) is parameter the statistical estimation of which, together with other parameters, is done based on time series of the variables \( Y(t), K, N \) and \( t \).

It should be noted that function (1) and the budget revenues function corresponding to it:

\[
T(t) = tY(t) = t\gamma DK^{\alpha(t)}N^{\beta(t)}, \quad (2)
\]

(or, on the whole, a model such as (1)–(2)) was developed by Balatsky for a broader purpose than that in connection with which we are talking about it in this case: for substantiating the macroeconomic concept of the Laffer curve and estimating the effect of fiscal policy on the level of business activity in a country with reasonable reliability [3: 89]. In spite of this, we believe that in modeling the relationship of the average tax rate and output, even with the version of the expanded production function (1), it is only partially possible to reflect the essence of the Laffer concept. The point is that the underlying essence of the Laffer theory – that is, its philosophy – consists in the idea that an increase or decrease in the tax burden, by creating a negative or positive system of stimuli, fosters a decline or growth in economic activity, which is primarily expressed in a change in the amount of use of resources, rather than in an increase or decrease in the efficiency of their use [4]. Consequently, to characterize the main aspect of the Laffer theory requires a model that is based on a behavioral equation and can reflect the positive and negative stimuli created by taxes, rather than a model based on the transformation equation (1), which for the most part is used to characterize production technology [5].

We can base the construction of this type of model on a generalized version of Arthur Laffer’s concept, according to which the aggregated (average) tax rate has an impact on total output in approximately the same form as on the amount of the budget’s tax rev-
enues [6, ch. 7]. Postulates of this concept can be formulated in a formalized way as follows:

1. At the extreme points \( t = 0 \) and \( t = 1 \) of the range of determination of the aggregated (average) tax rate, the values of total output \( Y(t) \) and budget revenues \( T(t) \) are equal to zero, that is: \( Y(0) = Y(1) = 0 \), \( T(0) = T(1) = 0 \);

2. There are values \( t^* \in [0, 1] \) and \( t^{**} \in [0, 1] \) of the average tax rate \( t \) such that \( Y(t) \) increases in the interval \( [0, t^*) \) and decreases in the interval \( (t^*, 1] \), and \( T(t) \) increases in the interval \( [0, t^{**}) \) and decreases in the interval \( (t^{**}, 1] \). In this case:

\[
\text{max}_{0 \leq t \leq 1} Y(t) = Y(t^*), \quad \text{max}_{0 \leq t \leq 1} T(t) = T(t^{**}).
\]

The average tax rate \( t^* \) at which output is maximum is called the Laffer fiscal point of the first kind, and \( t^{**} \) that produces the maximum budget revenues is called the Laffer fiscal point of the second kind. In the general case, these points are different from Balatsky’s points of the first and second kinds [3; 5]. It is clear that of the two points the more important one for the economy is the point of the first kind \( t^* \). Therefore, we arbitrarily call \( t^* \) the optimal tax rate.

Determination of the fiscal points \( t^* \) and \( t^{**} \) can be one of the conditions fostering improvement of a country’s economic policy. Two circumstances should be taken into account when constructing an appropriate model. The first one is that, in any economy, the total output depends on the amount and quality of existing economic resources (labor, capital, land, and production capabilities) and on the level of technology for using these resources. These factors determine the economy’s production-technology capabilities, and if they are distributed in the best possible way and fully used we have the maximum output, which is also called the potential output level. The second circumstance is that no less a role in the economy is played by the institutional environment, creation of which is a function of the government. Depending on how ideal the institutional environment is, in conditions of the same production-technology capabilities, the amount of output will be different for any two economies or any two periods of time. In the case of the best, that is, ideal, institutional environment, the actual and potential outputs are equal to each other. However, as a rule, the actually existing institutional environment differs from its ideal version in most cases. Therefore, the actual level of the economy’s total output is less than the potential level. Without question, an important role in creating the institutional environment is played by the tax system, along with a set of other factors. At the level of a model, we can set up a situation and assume that it is the tax system that is the main factor in creating the institutional environment that determines the behavior of economic agents. If we make such an assumption, then the total output function \( Y(t) \) can be represented in the following form [7]:

\[
Y(t) = Y_{pot} f(t), \quad (3)
\]

where \( Y_{pot} \) is the result expressing the economy’s production-technology capabilities; and \( f(t) \) is the function reflecting the institutional aspect.

From a formal point of view, \( Y_{pot} \) represents the maximum value of any macroeconomic production function in conditions of the optimal institutional environment. More specifically, \( Y_{pot} \) expresses the amount of potential output in conditions of the existing technology with full use of economic resources.

As for the function \( f(t) \) in (3), it describes the overall effect of taxes on total output. It is a behavioral function that, based on its content, should have the following properties:

1. \( f(t) \) is increasing in the interval \( [0, t^*) \) and decreasing in the interval \( (t^*, 1] \). In other words, from 0 to \( t^* \) an increase in the tax rate fosters an improvement of the institutional environment and growth in economic activity, while from \( t^* \) to 1 an increase in
the tax rate leads to deterioration of the institutional environment and a decline in economic activity;

2. For the optimal tax rate, \( f(t^*) = 1 \). This very important property indicates that the average tax rate \( t^* \) makes it possible to create an institutional environment in which the technological aspects of production completely determine the efficiency of output. Consequently, with the optimal average tax rate, output is maximum, and (3) takes the form:

\[
Y(t^*) = Y_{pot}
\]

3. It is desirable for \( f(t) \) to have one more property. In particular, in the absence of taxes, that is, when \( t = 0 \), \( f(0) = 0 \), while if the profit that is made is completely confiscated in the form of taxes, that is, if \( t = 1 \), then \( f(1) = 0 \). However, it should be noted that \( f(t) \) may not satisfy the third property, fully or partially. For example, for the case \( t = 0 \), \( f(t) \) will be different from zero if we suppose that there are state-owned firms and the government performs economic functions based on income received in the form of dividends from their profits.

We give an example of a total output function corresponding to (3), in which \( f(t) \) has the properties enumerated above. For this purpose, we use a modified version of the entropy function \((-t \ln t)[8; 9]:\)

\[
f(t) = -t^\delta \ln t^{e^\delta} . \tag{4}
\]

Then we have:

\[
Y(t) = Y_{pot} f(t) = Y_{pot} (-t^\delta \ln t^{e^\delta}) , \tag{5}
\]

where \( \delta \) is a statistically estimated positive parameter; and \( e \) is a Neperian number (base of the natural logarithm).

The budget revenues function corresponding to (5) has the following form:

\[
T(t) = tY(t) = tY_{pot} f(t) = Y_{pot} (-t^{\delta+1} \ln t^{e^\delta}) . \tag{6}
\]

It can be shown that, in the conditions of model (5)–(6), the values of the Laffer fiscal points of the first and second kind, \( t^* \) and \( t^{**} \), are determined as follows [10]:

\[
t^* = \exp\left(-\frac{1}{\delta}\right) = e^{-1/\delta} ,
\]

\[
t^{**} = \exp\left(-\frac{1}{(\delta+1)}\right) = e^{-1/(\delta+1)} . \tag{7}
\]

In addition, the following conditions are valid:

\[
\lim_{t \to 0} f(t) = 0 , \quad f(t^*) = 1 , \quad f(1) = 0 .
\]

Therefore, for the total output function (5) we have:

\[
\lim_{t \to 0} Y(t) = 0 , \quad Y(t^*) = Y_{pot} , \quad Y(1) = 0 .
\]

And for the budget revenues function (6):

\[
\lim_{t \to 0} T(t) = 0 , \quad T(t^{**}) = \frac{\delta}{1+\delta} Y_{pot} , \quad T(1) = 0 .
\]

As we see, in the conditions of model (5)–(6), the value of the fiscal characteristics \( t^* \) and \( t^{**} \) depend completely on parameter \( \delta \). To estimate the latter and, consequently, to identify model (5)–(6) we need observation data in relation to total output \( Y(t) \), the tax rate \( t \), and the potential output level \( Y_{pot} \). The last of these, \( Y_{pot} \), is not observable, that is, it is a latent quantity. Therefore, determining (estimating) its value requires developing a definite procedure, which is a separate problem.

To solve the problem involving \( Y_{pot} \), in model (5)–(7) we have to keep in mind that the potential output level \( Y_{pot} \) in contrast to the actual level, is determined by the amount of economic resources that exist but are not used. If we take only two aggregated resources into account – labor and capital – then we can write

\[
Y_{pot} = \varphi(\Phi, L) , \tag{8}
\]

where \( \Phi \) is the existing amount of capital; \( L \) is the existing amount of labor; and \( \varphi \) is some function.
that can be estimated, which can arbitrarily be called the technological function of potential output. This function cannot be estimated in isolation, by examining the expression \( Y_{pot} = \varphi(\Phi, L) \) only, since, as we pointed out above, we do not know the values of \( Y_{pot} \) in it. At the same time, if the value of \( Y_{pot} \) in the total output function (3) is replaced by the function \( \varphi(\Phi, L) \) and the expression obtained

\[
Y(t) = \varphi(\Phi, L) f(t)
\]

is transformed into a regression equation, then, along with \( f(t) \) we can also estimate \( \varphi(\Phi, L) \).

To illustrate this, we turn to statistical data that exist for the U.S. economy and consider 1970–2008 the period to be analyzed [11]. To determine the specific econometric form of (9), we represent the potential output function (8) as follows:

\[
Y_{pot(i)} = Ae^{\lambda i} L_{i-1}^\eta Y_{i-1}^\theta,
\]

where \( i \) is the time index; \( Y_{pot(i)} \) is potential output in period \( i \); \( A, \lambda, \mu, \eta, \) and \( \theta \) are parameters that can be statistically estimated; \( L_i \) and \( L_{i-1} \) are the amount of labor in periods \( i \) and \( i-1 \), respectively; and \( Y_{i-1} \) is actual output in period \( i-1 \).

Several circumstances determined the choice of such a structure for the potential output function. The first one involves overcoming the problem of autocorrelation. The lag variables (\( L_{i-1} \) and \( Y_{i-1} \)) are included in the model mostly for this purpose, although taking these variables into account expands the context of economic analysis, since it becomes possible to reflect dynamic aspects. The second circumstance involves reflection of the existing amount of capital. As we see, this factor of production, in contrast to labor, does not figure in the model in an explicit form. Calculations have shown that, if the amount of capital is taken into account, some of the model’s estimated parameters become statistically insignificant. Therefore, it is desirable to limit ourselves to just one basic factor: labor. What is more, even if there is no strictly econometric problem, it is justified to consider only labor as the main factor determining the potential output level. The point is that for the U.S. economy (and not only for it) labor is in shorter supply than capital. According to various calculations, for the United States the so-called natural capital utilization level is approximately 82 percent [12], while the natural rate of unemployment is less than 6 percent.

We incorporate (10) into (5), so that

\[
Y_i(t) = Y_{pot(i)} f(t) = Ae^{\lambda i} L_i^\mu L_{i-1}^\eta Y_{i-1}^\theta (-i_1^\delta \ln t_1^\delta),
\]

and by taking the logarithm we transform this expression into a regression equation with the following form:

\[
\ln \left( \frac{Y_i(t)}{-e^{\ln t}} \right) = \ln(A\delta) + \lambda i + \mu \ln L_i + \\
+ \eta \ln L_{i-1} + \theta \ln Y_{i-1} + \delta \ln t_i + \ln e_i,
\]

where \( e_i \) is a random term characterizing the part of the actual output’s deviation from the potential level that is determined by so-called nontax circumstances. The results of estimation of this equation are given in Table 1. As we see, all of the model’s estimated coefficients and the absolute term are statistically significant. The regular and adjusted coefficients of determination are highly significant. And there is no autocorrelation problem (it is denied by the Durbin h-test, at both the 5 percent and 1 percent significance levels). Consequently, the estimated model is suitable for drawing certain conclusions.

Based on the \( \delta \) given in Table 1, using equations (7), it is easy to establish that for the period being analyzed: \( t^* = 0.316, t^{**} = 0.586 \).

This result indicates several interesting things to us.

First, the derived value of the Laffer fiscal point of the first kind, that is, the optimal tax rate \( t^* \) is somewhat higher than the actual value of \( t \) for each year over the course of the period under consideration (for reference, the actual values of \( t \) in 1970–2008 satisfied the inequality \( 0.261 \leq t \leq 0.303 \), and the average value of \( T \) for the period was 0.277). We can
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Table 1. Results of Estimation of Regression Equation (12)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Estimates</th>
<th>Standard Errors</th>
<th>Student’s t -Test</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONS</td>
<td>ln(4δ)</td>
<td>4.1663</td>
<td>1.1740</td>
<td>3.5486</td>
<td>0.0012</td>
</tr>
<tr>
<td>t</td>
<td>λ</td>
<td>0.0168</td>
<td>0.0042</td>
<td>4.0091</td>
<td>0.0003</td>
</tr>
<tr>
<td>lnL_t</td>
<td>μ</td>
<td>2.2793</td>
<td>0.5945</td>
<td>3.8342</td>
<td>0.0005</td>
</tr>
<tr>
<td>lnL_{t-1}</td>
<td>η</td>
<td>-2.1935</td>
<td>0.5703</td>
<td>-3.8465</td>
<td>0.0005</td>
</tr>
<tr>
<td>lnY_{t-1}</td>
<td>θ</td>
<td>0.4334</td>
<td>0.1259</td>
<td>3.4435</td>
<td>0.0016</td>
</tr>
<tr>
<td>lnT_t</td>
<td>δ</td>
<td>0.8685</td>
<td>0.0839</td>
<td>10.3453</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R² = 0.9985, adjusted R² = 0.9982, F(4;34) = 4390, p < 0.0000; DW = 1.577, h = 1.65

t** differ significantly from each other: according to the results obtained, t** is 0.586, almost twice as great as t*. At the model level, we can determine what would have happened with the U.S. economy in the period under consideration if the average tax rate had been raised from the actual average value for the period to 0.586. According to the model, the average tax rate for period $T = 0.277$ corresponds to a lag of approximately 0.7 percentage points behind the potential output. All else being equal, increasing the tax rate to 0.586 would have increased this lag to 20 percent. This casts doubt on the advisability of an economic policy in which the government’s priority is to maximize the budget’s tax revenues.

We consider it necessary to make one extremely important clarification. We have in mind that the deviation of the actual output from the potential output may be caused by the effect of a nonoptimal tax burden or by other, nontax circumstances and factors. The function $(1 - f(t))$, that is, the percent difference between the potential and actual outputs. A graphic illustration of this is given in Figure 1, which shows the dynamics of percent values of the lost gross domestic product due to nonoptimality of the tax burden. Figure 1 shows that, according to model (5)–(6), if the Laffer theory is correct, there was a certain resource in the U.S. economy for increasing output by optimizing the tax burden. Because of the low tax burden, this resource was greater than 1 percent in some years (1971, 1975, 1983, 1984, and 2003), and less than 0.2 percent in other years. If we calculate the average value during the period, we find that in 1970–2008 the average annual lag of actual output behind the optimal level due to a nonoptimal tax burden was 0.66 percent. This is a considerable reserve, and therefore it may be said that during the period under consideration, on average, the U.S. economy functioned in conditions of a nonoptimal tax burden.

Second, no less attention should be given to the circumstance that the Laffer fiscal points $t^*$ and $t^{**}$ differ significantly from each other: according to the results obtained, $t^{**}$ is 0.586, almost twice as great as $t^*$. At the model level, we can determine what would have happened with the U.S. economy in the period under consideration if the average tax rate had been raised from the actual average value for the period to 0.586. According to the model, the average tax rate for period $T = 0.277$ corresponds to a lag of approximately 0.7 percentage points behind the potential output. All else being equal, increasing the tax rate to 0.586 would have increased this lag to 20 percent. This casts doubt on the advisability of an economic policy in which the government’s priority is to maximize the budget’s tax revenues.

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Fig. 1. Dynamics of deviation of actual output level behind optimal level due to nonoptimality of the tax burden in the United States in 1970-2008.

Fig. 2. Dynamics of deviation of actual output from the potential output level and unemployment in the United States in 1970-2008 (in %)

According to model (12). As the figure shows, in individual years the deviation from the potential was three or more percentage points, while the maximum deviation because of the nonoptimal tax burden was approximately 1.2 percent. Moreover, in individual years the effect of nontax circumstances was so strong that it exceeded the negative stimuli due to nonoptimality of the tax burden, and the actual output, instead of lagging behind, exceeded the potential output. In Figure 2, such cases correspond to negative values of the deviation.

Along with the curve for the dynamics of deviations of actual output from the potential output, Figure 2 also shows the curve for the dynamics of the actual unemployment rate. As we see, the movements of these two curves are very similar to each other, which indicates that in conditions of high unemployment the lag behind potential output was accordingly high, while in the case of especially low unemployment rate (less than 6 percent), actual output exceeded the potential. This result should be especially emphasized, since in the proposed model nei-
ther the unemployment rate nor the amount of labor used are entered as exogenous variables. Moreover, model (5)–(6) makes it possible to endogenously estimate the value of the natural rate of unemployment (the level of unemployment existing in the conditions of potential output).

In summary, we note that the model considered here for estimating the effect of the tax burden on the amount of resource use has worked fairly well in regard to data on the U.S. economy. The results obtained are entirely plausible in an economic sense. When different versions of the calculations were carried out, the estimated model as a whole, as well as its parameters, maintained its stability and did not lose its statistical significance in a fairly broad range of changes in the “sample size.” It is interesting that, even when the quality of the model deteriorated (the parameters being estimated became statistically insignificant) as a result of excessive reduction of the sample size, the estimates of the fiscal characteristics $t^*$ and $t^{**}$ changed only slightly. Naturally, all of this is not sufficient grounds for drawing final conclusions about the suitability of the suggested model for conducting specific applied calculations. However, we do believe that, after some future improvements and testing of its performance with statistical data from various countries, the suggested approach may prove to be perfectly acceptable for estimating the efficiency of fiscal policy.
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